Basic Mathematical Model for Spread of Covid-19 Disease

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India's top health research body, The Indian Council of Medical Research (ICMR) has initiated a study that is expected to predict the rate of Covid-19 infections in the country in the coming months. ICMR will use mathematical modeling to figure out how many cases India could be grappling with in the next few months. Unlike other countries, where older population is more venerable to the disease, 60 % of the cases in India are between 20 to 49 years!

DR. R R Gangakhedkar, chief epidemiologist at ICMR, said that the research body through the modeling process will know the best-case and worst-case scenarios. "Epidemiologists conducting mathematical modeling of the SARS-CoV-2 virus will help estimate the transmission rates of the Covid-19 virus, as well as the variance of transmission rates over time, he said. ICMR initiates study to predict the rate of Covid-19 infections in India [1]

Mathematical studies of infectious diseases which are spread by direct person-to-person contact in a population are carried through basic types of deterministic models. Here, this simplest model is formulated as the initial value problems for systems of ordinary differential equations and are analyzed mathematically.

The models, presented here are simple and their mathematical analyses are elementary. But these models provide notation, concepts, intuition and foundation for considering more refined models. In this model, the population of city, region or country under consideration is divided into disjoint classes which change with time t.[2]

S=S(t) The susceptible class, consists of those individuals who can incur the disease but are not yet infective. The I=I(t), the infective class consists of those who are transmitting the disease to others. The R=R(t), removed class consists of those who are removed from the susceptible-infective interaction by recovery with immunity, isolation, or death.

SIR model for Spread of Disease:

As the first step in the modeling process, we identify the independent and dependent variables. The independent variable is time t, measured in days. We consider two related sets of dependent variables. The first set of dependent variables, counts *people* in each of the groups, each as a function of time:' t'

S = S(t)	is the number of *susceptible* individuals,
I = I(t)	is the number of *infected* individuals, and
R = R(t)	is the number of *removed* individuals.(due to recovery or death)

In this model we need following assumptions.

We assume that the epidemic is short and total population i.e. S(t) +I(t) +R(t) = N remains constant.

No one is *added* to the susceptible group, since we are ignoring births and immigration. The only way an individual *leaves* the susceptible group is by becoming infected.

We assume that the time-rate of change of S(t) i.e , depends on the number already susceptible, the number of individuals already infected, and the rate of contact(transmission rate) 'r' between susceptible and infected.

Thus, on average, each infected individual generates r S(t) new infected individuals per day.

We also assume that a rate of change of R(t) is constant 'a'[3]

Now under these assumptions, equations governing mathematical model are

$\frac{dS}{dt} = -rIS \ldots\ldots..(i)$ (Minus sign because, S is decreasing.)

$\frac{dI}{dt} = rIS - aI \ldots\ldots..(ii)$

$\frac{dR}{dt} = aI \ldots\ldots..(iii)$

We use initial conditions as $S = S_0$, $I = I_0$, and $R = 0$ at $t = 0$.

Also we have $S(t) + I(t) + R(t) = N$ i.e. $\frac{d}{dt}(S + I + R) = 0$.

i.e. $S + R + I = S_0 + I_0$.

Eq. (i), indicates that the rate of change of S is negative, i.e. S is always decreasing

i.e. $S \ll S_0$ From equation (ii) we observe that $\frac{dI}{dt} < I(rS_0 - a)$.

Now from this equation we can discuss whether decease will spread or not? Sign of $(rS_0 \; - a)$is important and deciding factor . Now it can be easily observed that if the value of I increases than its initial value Io, then the decease spreads! That is $(rS_0 \; - a)$>0 i.e S_0 >a/r. [4]

This a/r = q is the contact ratio. Further $R_0 = \frac{rS_0}{a}$ represents basic secondary infection in the population caused by one primary infection. The basic reproductive number, R_0, is the number of secondary infections that one infected person would produce in a fully susceptible population through the entire duration of the infectious period. R_0 provides a threshold condition for the stability of the disease-free equilibrium point (for most models) is 1.

The disease-free equilibrium point is locally, asymptotically stable when $R_0 < 1$: the disease dies out. The disease-free equilibrium point is unstable when $R_0 > 1$: the disease establishes itself in the population or an epidemic It should be noted that R_0 >1 if we have epidemic. In case of seasonal flew $1.5 < R_0 < 2$. For COVID 19 it is estimated to be between 2 and 4 i.e. $2 < R_0 < 4$.

Now we try to answer the question, what is the maximum number of infectives at a given time? Answer to this question will help the agencies to plan strategies for controlling the decease.

From eqs. (i) & (ii), we obtain $\frac{dI}{dS} = \frac{rIS-aI}{-rIS}$ = -1 + a/rS = -1 + 1/qS (as q=r/a)

On solving this diff.eq. we get $I + S - (1/q) \log S = I_0 + S_0 - (1/q) \log S_0$

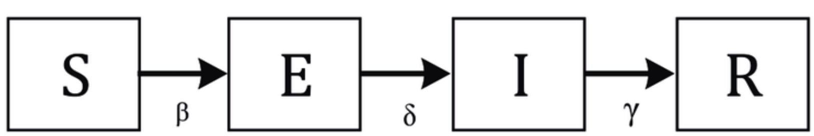
$I_{max} = I_0 + S_0 - (1/q)\,(1+\log(qS_0)) = I_0 + S_0 - f(x)$ where $f(x) = 1/q(1+\log(qS_0))$

It can be seen that, though we cannot stop the spread of the decease but I_{max} can be reduced by making f(q) as large as possible. This happens when q is small. Thus reducing the value of q, the contact ratio is our aim to control the COVID-19. That's why washing your hands, social distancing etc are the available remedies to control the decease.[5]

SEIR Model:

It is well known that the stochastic model is more accurate in evaluating real-life epidemic propagation. But, when the population is large enough, simpler deterministic model turns to be good enough to use. Of the many deterministic models, the Susceptible- Exposed-Infectious-Removed (SEIR) models are often implemented when studying the spread of infectious diseases that possess significant incubation periods. In the case of COVID-19 several studies from China have also used this SEIR model.

In this study we partitions the population into S, I and R compartments but with an additional 'exposed' (E) state between the S and I states. Such models are called SEIR models. Among those infected, some fraction may be quarantined if they can be identified. If infected individuals are not identified, they can infect susceptible people before they themselves recover or die. Those who have been quarantined can't infect anyone.



The infectious rate, β, controls the rate of spread which represents the probability of transmitting disease between a susceptible and an infectious individual. The incubation rate, δ, is the rate of latent individuals becoming infectious (average duration of incubation is 1/δ). Recovery rate, ɣ = 1/D, is determined by the average duration, D, of infection. The differential equations governing SEIR model are:

$$\frac{dS}{dt} = -\frac{\beta SI}{N}$$

$$\frac{dE}{dt} = -\frac{\beta SI}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

The SEIR model is based on an estimation of reproduction rate of the virus, The report also predicted two scenarios to show when the case tally could fall to zero In the first scenario, if the lockdown is extended to May 15, the case tally will fall to zero by September 15. In the second scenario, if the lockdown is extended to May 30, cases will drop to zero by mid-June.[6]

Data for **India:** (21/5/20 at 8:26pm) Confirmed: 1,14,478, Active:64,545, Recovered: 46,461, Deceased: 3,465.

Maharashtra : Conformed:39,297 Active:27,589 Recovered: 10,318 Deceased:1,390.

Fatality rate 3.22.[6]

Despite concerns, that the relaxations on mobility and businesses would yield a spike in infection numbers, the nation's R_0 has continued its downward trend, falling to 1.81 as of May 15, three days before the implementation of the fourth phase of the lockdown. According to data compiled by the Times Fact India Outbreak Report, India's R_0 stood at 1.99 as of April 20, around six days into the second phase of the lockdown. Over the next two weeks, it began trending downward, dropping to 1.9 on May 4, the date on which the third phase of the lockdown came into effect.

Maharashtra - the worst-affected state - which had a high R_0 of 2.19 on April 26 has managed to significantly reduce this figure in the last three weeks, with our data pegging it at 1.76 on May 15. [7]

Semilog plot of the spread of SARS-CoV-2 and of COVID-19 recoveries & deaths in Maharashtra

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Conclusion:

At the current growth rate of epidemic, India's healthcare resources will be overwhelmed by the end of May. With the immediate institution of NPIs, total cases, hospitalizations, ICU requirements, and deaths can be reduced by almost 90%. Uninterrupted epidemic in India would have resulted in more than 364 million cases and 1.56 million deaths with peak by mid-July. As per the model, at current growth rate of 1.15, India is likely to reach approximately 3 million cases by 25 May, implying 125,455 (±18,034) hospitalizations, 26,130 (±3298) ICU admissions, and 13,447 (±1819) deaths. This would overwhelm India's healthcare system. The model shows that with immediate institution of NPIs, the epidemic might still be checked by mid-April 2020.

It would then result in 241,974 (±33,735) total infections, 10,214 (±1649) hospitalizations, 2121 (±334) ICU admissions, and 1081 (±169) deaths.[9]

India is projected to hit a peak of 1,91,697 active cases according to the most likely model and 2,77,613 active cases according to the SEIR model, on June 18. Maharashtra is projected to hit a peak of 55,545 active cases according to the most likely model and 66,814 active cases according to the SEIR model, on June 10. The SEIR model describes the effects of a lockdown by changing parameters to represent a total 'switching off' of infections. They find that a single lockdown of 21 days has little effect, at least beyond a temporary suppression of the case growth rate. Instead, it recommends a single 48-day lockdown for a more long-lasting effect.

REFERANCES:

1] https://economictimes.indiatimes.com/industry/healthcare/biotech/healthcare/icmr-initiates-study-to-predict-the-rate-of-covid-19-infections-in-india/articleshow/74768015.cms?utm_source=contentofinterest&utm_medium=text&utm_campaign=cppst

2] http://www.mtholyoke.edu/~ahoyerle/math333/ThreeBasicModels.pdf

3] https://www.youtube.com/watch?v=NKMHhm2Zbkw3

4] https://www.youtube.com/watch?v=NKMHhm2Zbkw

5] https://www.youtube.com/watch?v=NKMHhm2Zbkw

[6] https://www.covid19india.org/]

[7] https://timesofindia.indiatimes.com/life-style/health-fitness/health-news/epidemic-could-peak-in-india-by-mid-may-study/articleshow/75287335.cms

[8] Med J Armed Forces India. 2020 Apr 2doi: 101016. /j.mjafi.2020.03.022